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HYDROMAGNETIC STAGNATION-POINT BOUNDARY LAYER WITH ARBITRARY PRESSURE GRADIENT AND MAGNETIC FIELD

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SUMMARY

The series-expansion method of H. Görtler is applied to the two-dimensional stagnation flow about symmetric bodies with symmetric distributions of magnetic field. Numerical computations are presented from which skin friction and heat transfer can be calculated through third order in a modified surface coordinate provided the external velocity, magnetic field, and enthalpy are known. A small magnetic Reynolds number is assumed; the Prandtl number is taken to be unity. The three chosen values of an interaction parameter represent magnetic fields in which the component normal to the wall has values at the stagnation point that are zero, small, and large.

INTRODUCTION

The discussions of the hydromagnetic flow at a two-dimensional stagnation point presented in the literature have in each case been similarity treatments. With the velocity outside the boundary layer varying as the distance from the stagnation point, the normal component of the magnetic field is constant. In reference 1 it is pointed out that the skin friction was markedly reduced by the action of the normal magnetic field. Later investigations (refs. 2 to 4) showed that the effect upon heat transfer was not as strong. Reference 5 treats this problem when the fluid is compressible, while reference 6 discusses the corresponding axisymmetric boundary layer.

In this report the restriction to similarity will be relaxed so that stagnation flows around symmetric, blunt bodies of otherwise arbitrary shape and with symmetric but otherwise arbitrary distributions of magnetic fields may be studied. For this purpose, the series-expansion method of Görtler (refs. 7 and 8) will be extended to flows with magnetic fields present. In addition to its ability to handle arbitrary pressure gradients, the Görtler method has the feature, in contrast to the usual expansion in the streamwise coordinate, that the first term (zero order) of the series solution matches the outer velocity at the edge of the boundary layer at all stations along the body. Subsequent terms represent corrections to the velocity profile that are due to action within the boundary layer itself. Although the series does converge rapidly, Görtler found that, for stagnation-point flows, this series was not more advantageous than the well-known Blasius series; however, for the present purpose of extension to magnetic effects, the Görtler series has the advantage that the nonmagnetic boundary-layer terms have already been computed to a high degree of precision. This is a convenience in undertaking further numerical computations.

Several extensions of Görtler's work have already been made. The temperature has been developed in a Görtler series (refs. 9 and 10), and the case of the flow over a flat plate with a current-carrying wire near its surface has been solved in reference 11. The developments discussed in this report are all based on the assumption that the magnetic Reynolds number is small.

SYMBOLS

B	magnetic field
b	dimensionless magnetic field
c_p	heat capacity
D_n	differential operator
E	electric field
$F(\xi,\eta)$	reduced stream function
$f_{i,j}, f_{i,jk}, f_{ij,k}, f_{0,ijk}, f_{ijk,0}$	higher order stream functions, where $(i,j,k) = (0,1,2,3)$
$g_{i,j}, g_{i,jk}, g_{ij,k}, g_{0,ijk}, g_{ijk,0}$	higher order enthalpy functions, where $(i,j,k) = (0,1,2,3)$
H	dimensionless enthalpy difference
h	enthalpy
k	heat conductivity
L	characteristic length
L_n	differential operator in eq. (34)
P	pressure
Pr	Prandtl number

p	dimensionless pressure
q	heat transfer
R_h	magnetic pressure number
R_{h0}	magnetic pressure number for symmetric magnetic field, defined by eq. (65)
R_{h1}	magnetic pressure number for antisymmetric magnetic field, defined by eq. (62b)
R_m	magnetic Reynolds number
R_{m1}	magnetic Reynolds number based on stream-velocity gradient at $X = 0$, defined by eq. (62a)
Re	Reynolds number
Re_1	Reynolds number based on stream-velocity gradient at $X = 0$, defined by eq. (56)
T	temperature
t	time
U,V	velocity components in X- and Y-directions
U_e	velocity external to boundary layer
$\vec{U}$	velocity
u,v	dimensionless velocity components
$\vec{u}$	dimensionless velocity
X,Y	coordinates along and normal to body surface
$\beta(\xi)$	principal function, defined by eq. (27a)
δ	boundary-layer thickness
η	boundary-layer coordinate
$\lambda(\xi)$	magnetic interaction coefficient, defined by eq. (27b)
μ	magnetic permeability
ν	kinematic viscosity
ξ	coordinate in streamwise direction

ρ	density
σ	electrical conductivity
τ	skin friction
ψ	stream function
ω	vorticity

Subscripts:

o	reference quantity
X,Y	vector components
ξ,η	partial derivatives
0,1,2, . . .	coefficients in a series expansion

Superscripts:

'	first derivative
"	second derivative
^	intermediate variable

FLOWS AT SMALL MAGNETIC REYNOLDS NUMBER

A body, immersed in a conducting fluid, from which there emanates a magnetic field is considered. The fluid has constant electrical conductivity, density, viscosity, thermal conductivity, and heat capacity. Under the further restrictions of steady velocity field and steady magnetic field, the governing equations for the motion of the fluid are

$$\nabla \cdot \vec{U} = 0 \tag{1}$$

$$\vec{U} \cdot \nabla\vec{U} = -\frac{1}{\rho}\nabla P + \nu\nabla^2\vec{U} + \frac{\sigma}{\rho}(\vec{E} + \vec{U} \times \vec{B}) \times \vec{B} \tag{2}$$

The action of the velocity field upon the magnetic field is described by the equation

$$\nabla^2\vec{B} = -\mu\sigma\nabla \times (\vec{U} \times \vec{B}) \tag{3}$$

One important restriction placed upon equations (2) and (3) is the representation of the electrical conductivity as a scalar σ. If the fluid is a gas, it must be sufficiently dense for the mean free path of a charged particle to be much smaller than the radius of gyration in the magnetic field. Equations are written in mks units.

In order to find the dimensionless parameters that characterize the various forces involved in the motion, a characteristic velocity U_o, a characteristic value of the magnetic field B_o, and a characteristic length L, which is assumed to characterize both the body and the magnetic field, are used to nondimensionalize. The aforementioned assumption is justified if the currents generating the magnetic field are not too close to the body surface; if they are near the body center, the radius of curvature of the field at the body surface must be comparable to the radius of curvature of the body. If the dimensionless dynamic variables are denoted by lower-case letters, the governing equations (eqs. (1) to (3), respectively,) become

$$\nabla \cdot \vec{u} = 0 \tag{4}$$

$$\vec{u} \cdot \nabla\vec{u} = -\nabla p + \frac{1}{Re} \nabla^2\vec{u} + R_m R_h \left(\frac{\vec{E}}{U_o B_o} + \vec{u} \times \vec{b}\right) \times \vec{b} \tag{5}$$

$$\nabla^2\vec{b} = -R_m \nabla \times (\vec{u} \times \vec{b}) \tag{6}$$

The parameters that appear are the magnetic pressure number $R_h \equiv B_o^2/\mu\rho U_o^2$ and the magnetic Reynolds number $R_m \equiv \mu\sigma U_o L$, which is common to the interaction terms of equations (5) and (6). In the following work R_m is considered to be small, but at this point no restriction is placed on R_h. It is appropriate, then, to write the series (ref. 12):

$$\left.\begin{aligned} \vec{u} &= \vec{u}_0 + R_m\vec{u}_1 + R_m^2\vec{u}_2 + \ldots \\ \vec{b} &= \vec{b}_0 + R_m\vec{b}_1 + R_m^2\vec{b}_2 + \ldots \\ p &= p_0 + R_m p_1 + R_m^2 p_2 + \ldots \end{aligned}\right\} \tag{7}$$

The electric field E is the field measured by an observer at rest with respect to the body. In general, the electric field is the result both of any external generators and of the separation of charges by the electromotive force $\vec{u} \times \vec{b}$. While the first of these is excluded, the second effect may arise either from the polarization of molecules or from the separation of free charges so that the fluid is not electrically neutral everywhere. The permittivity of the fluid is assumed to be constant everywhere, and the flow is assumed to be truly two-dimensional with no end effects. Then E may be set equal to 0 because the fluid is neutral and because the polarization field, when added to the electromotive force, reduces the effective conductivity by the same factor over the entire field. Another justification for choosing $E = 0$ is to consider the two-dimensional geometry as the limit of an axisymmetric geometry as the radius becomes very large. Inasmuch as the electric potential must be continuous along a circular path about the axis of symmetry, the azimuthal component of the electric field must vanish (ref. 13).

Equations (4) to (6), respectively, become to lowest order

$$\nabla \cdot \vec{u}_0 = 0 \tag{8}$$

$$\vec{u}_0 \cdot \nabla\vec{u}_0 = -\nabla p_0 + Re^{-1}\nabla^2\vec{u}_0 + R_m R_h(\vec{u}_0 \times \vec{b}_0) \times \vec{b}_0 \tag{9}$$

$$\nabla^2\vec{b}_0 = 0 \tag{10}$$

According to equation (10), the magnetic field is unaffected by the motion of the fluid to the order indicated, but the velocity field will, in general, be disturbed by the magnetic field because the range of R_h is unrestricted and the product $R_m R_h$ may be of order unity or greater.

BOUNDARY-LAYER ANALYSIS

The dimensional momentum equation for the fluid and the modifications resulting from the assumption of a viscous boundary layer shall be considered now. In a two-dimensional geometry let the X-coordinate be along the body while the Y-coordinate is normal to the body. The corresponding components of velocity are U and V. The components of the Lorentz force are proportional to

$$-UB_Y^2 + VB_X B_Y \tag{11a}$$

and

$$-VB_X^2 + UB_X B_Y \tag{11b}$$

If δ is characteristic of the boundary-layer thickness, then

$$\frac{V}{U} \sim \frac{\delta}{L} \ll 1 \tag{12}$$

Consequently, the force terms reduce to

$$-UB_Y^2 \tag{13a}$$

and

$$+UB_X B_Y \tag{13b}$$

In the inertia and viscous forces certain terms also drop out, because the ratio of boundary-layer thickness to body curvature δ/L is small. The final form of the momentum equation is

$$U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = \frac{-\sigma}{\rho} UB_Y^2 - \frac{1}{\rho}\frac{\partial P}{\partial X} + \nu \frac{\partial^2 U}{\partial Y^2} \tag{14}$$

$$0 = \sigma UB_X B_Y - \frac{\partial P}{\partial Y} \tag{15}$$

From equations (14) and (15) it follows that

$$\frac{\partial P}{\partial Y} \sim R_m R_h \tag{16}$$

As long as this product remains of order unity or less, it is permissible to replace the streamwise pressure gradient in the boundary layer with the streamwise pressure gradient in the external stream, for then the pressure increment across the boundary layer is $\mathcal{O}(\delta/L)$ compared to the pressure drop along the body. The pressure gradient is obtained by writing equation (14) at the edge of the boundary layer

$$U_e \frac{\partial U_e}{\partial X} = \frac{-\sigma}{\rho} U_e B_Y^2 - \frac{1}{\rho}\frac{\partial P}{\partial X} \tag{17}$$

In equation (17) the value of B_Y at the edge of the boundary layer may be equated to the value at the wall. Subtracting equation (17) from equation (14) results in a form of the boundary-layer equation that is free of the pressure gradient:

$$U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} - \frac{\sigma}{\rho} B_Y^2 U_e \left(1 - \frac{U}{U_e}\right) - U_e \frac{dU_e}{dX} = \nu \frac{\partial^2 U}{\partial Y^2} \tag{18}$$

The boundary conditions to be used with this equation are that U vanish at the body surface and approach the velocity of the inviscid stream at the outer edge of the boundary layer. The second of these must be used with care. The interaction of the magnetic field and the velocity field produces a vorticity in the inviscid stream. If the vorticity in the inviscid stream approaches in magnitude the vorticity due to viscosity in the boundary layer, it is necessary to augment the boundary condition at the outer edge of the boundary layer by a matching of the velocity derivatives. In a two-dimensional incompressible flow, the vorticity production is given by

$$\frac{\partial \vec{\omega}}{\partial t} + U \cdot \nabla\vec{\omega} = \nu\nabla^2\vec{\omega} + \frac{\sigma}{\rho}\vec{B} \cdot \nabla(\vec{U} \times \vec{B}) \tag{19}$$

The first term on the right represents the generation of vorticity by viscous action in the boundary layer; its approximate magnitude is $\nu U_e/\delta^3$. The second term is the vorticity production by the Lorentz force, which has an approximate magnitude $\sigma U_e B^2/\rho L$. The two lengths appearing are related by the Reynolds number:

$$(L/\delta)^2 \sim Re$$

It follows that

$$\frac{\text{Vorticity production in inviscid flow}}{\text{Vorticity production in boundary layer}} \sim \frac{R_m R_h}{\sqrt{Re}} \tag{20}$$

As long as

$$R_m R_h \ll \sqrt{Re} \tag{21}$$

it is permissible to match boundary-layer and inviscid-flow velocities at the edge of the boundary layer. Otherwise, it is necessary to use the boundary condition involving velocity derivatives. The restriction (21) on R_h is generally not as severe as the previous restriction (16). It is assumed in the following discussion that the ratio (20) is small.

APPLICATION OF GÖRTLER TRANSFORMATION

Following Görtler (ref. 7) the boundary-layer equation (eq. (18)) will be transformed to a system of ordinary differential equations. First, there are defined the dimensionless variables

$$\left.\begin{aligned} \hat{\xi} &= \frac{1}{\nu}\int_0^X U_e(X)\,dX \qquad & \hat{\eta} &= \frac{1}{\nu}U_e(X)Y \\ \hat{u} &= \hat{\psi}_{\hat{\eta}} = \frac{U}{U_e} \qquad & \hat{v} &= -\hat{\psi}_{\hat{\xi}} = U_e^{-2}(U_eV + U_e'YU) \end{aligned}\right\} \tag{22}$$

Substituting these variables in equation (18) results in

$$\hat{u}\hat{u}_{\hat{\xi}} + \hat{v}\hat{u}_{\hat{\eta}} - \hat{\beta}(\hat{\xi})(1 - \hat{u}^2) - \hat{\lambda}(\hat{\xi})(1 - \hat{u}) = \hat{u}_{\hat{\eta}\hat{\eta}} \tag{23}$$

where

$$\left.\begin{aligned} \hat{\beta}(\hat{\xi}) &\equiv \frac{\nu U_e'}{U_e^2} \\ \hat{\lambda}(\hat{\xi}) &\equiv \frac{\sigma\nu B_Y^2}{\rho U_e^2} \end{aligned}\right\} \tag{24}$$

Next, the Blasius transformation

$$\left.\begin{aligned} \xi &= \hat{\xi} \\ \eta &= \frac{\hat{\eta}}{\sqrt{2\hat{\xi}}} \\ \hat{\psi}(\hat{\xi},\hat{\eta}) &= \sqrt{2\hat{\xi}}\,F(\xi,\eta) \end{aligned}\right\} \tag{25}$$

is employed in equation (23) to obtain

$$F_{\eta\eta\eta} + FF_{\eta\eta} + \beta(\xi)\left(1 - F_\eta^2\right) + \lambda(\xi)(1 - F_\eta) = 2\xi(F_\eta F_{\xi\eta} - F_\xi F_{\eta\eta}) \tag{26}$$

where

$$\beta(\xi) \equiv 2\xi\hat{\beta}(\hat{\xi}) \tag{27a}$$

$$\lambda(\xi) \equiv \frac{2\xi\sigma\nu B_Y^2}{\rho U_e^2} \tag{27b}$$

Equation (26) is Görtler's equation for a boundary layer in an arbitrary pressure gradient with the addition of the term containing $\lambda(\xi)$, which represents the influence of the magnetic field on the flow. The parameter $\lambda(\xi)$ combines the effects of the magnetic field distribution and the outer velocity distribution.

The boundary conditions associated with equation (26) have also been developed by Görtler (ref. 12). The requirement that the stream function vanish on the surface is

$$F(\xi,o) = 0 \tag{28a}$$

The condition of no slip (U = 0) on the surface becomes

$$F_\eta(\xi,o) = 0 \tag{28b}$$

and the matching condition at the edge of the boundary layer, $U = U_e$, transforms to

$$F_\eta(\xi,\infty) = 1 \tag{28c}$$

A system of ordinary differential equations can be derived from equation (26) by first expanding the reduced stream function $F(\xi,\eta)$ and the parameters $\lambda(\xi)$ and $\beta(\xi)$ in power series in ξ:

$$\left.\begin{aligned} \beta(\xi) &= \beta_0 + \beta_1\xi + \ldots \\ \lambda(\xi) &= \lambda_0 + \lambda_1\xi + \ldots \\ F(\xi,\eta) &= F_0(\eta) + F_1(\eta)\xi + F_2(\eta)\xi^2 + \ldots \end{aligned}\right\} \tag{29}$$

When equations (29) are substituted into equation (26) and coefficients of like powers of ξ are equated, an ordinary differential equation in η for each of the functions $F_0(\eta)$, $F_1(\eta)$, . . . results. The expansion coefficients (β_0, β_1, . . . and λ_0, λ_1, . . .) depend upon the external velocity distribution

$$U_e = U_0 + U_1X + \ldots \tag{30}$$

and the distribution of normal magnetic field at the surface

$$B_Y = B_0 + B_1X + \ldots \tag{31}$$

The coefficients β_n and λ_n for symmetric blunt bodies and magnetic fields are listed in a later section.

The equation governing the zero-order stream function is

$$F_0''' + F_0F_0'' + \beta_0\left(1 - F_0'^2\right) + \lambda_0(1 - F_0') = 0 \tag{32}$$

The associated boundary conditions are

$$\left.\begin{aligned} F_0(0) = F_0'(0) &= 0 \\ F_0'(\infty) &= 1 \end{aligned}\right\} \tag{33}$$

A differential operator appearing in the equations for the subsequent F_n is defined by

$$L_nF_n \equiv F_n''' + F_0F_n'' - 2(\beta_0 + n)F_0'F_n' - \lambda_0F_n' + (2n + 1)F_0''F_n \tag{34}$$

Then

$$\left.\begin{aligned} L_1F_1 &= \beta_1(F_0'^2 - 1) - \lambda_1(1 - F_0') \\ L_2F_2 &= \beta_2(F_0'^2 - 1) + 2\beta_1F_0'F_1' + (\beta_0 + 2)F_1'^2 - 3F_1F_1'' \\ &\quad - \lambda_2(1 - F_0') + \lambda_1F_1' \\ L_3F_3 &= \beta_3(F_0'^2 - 1) + \beta_1F_1'^2 + 2\beta_1F_0'F_2' + 2\beta_2F_0'F_1' \\ &\quad + 2(\beta_0 + 3)F_1'F_2' - 3F_1F_2'' - 5F_2F_1'' - \lambda_3(1 - F_0') + \lambda_1F_2' + \lambda_2F_1' \end{aligned}\right\} \tag{35}$$

The associated boundary conditions are

$$F_n(0) = F_n'(0) = F_n'(\infty) = 0 \tag{36}$$

In order to solve equations (35) and (36) in terms of functions independent of the β's and λ's, it is convenient to resolve the F_n as follows:

$$F_1 = \beta_1f_{1,0} + \lambda_1f_{0,1}$$

$$F_2 = \beta_2f_{2,0} + \beta_1^2f_{11,0} + \beta_1\lambda_1f_{1,1} + \lambda_1^2f_{0,11} + \lambda_2f_{0,2}$$

$$\begin{aligned} F_3 &= \beta_3f_{3,0} + \beta_1^3f_{111,0} + \beta_1^2\lambda_1f_{11,1} + \beta_1\lambda_1^2f_{1,11} + \beta_1\beta_2f_{12,0} \\ &\quad + \beta_1\lambda_2f_{1,2} + \beta_2\lambda_1f_{2,1} + \lambda_1^3f_{0,111} + \lambda_1\lambda_2f_{0,12} + \lambda_3f_{0,3} \end{aligned}$$

In these equations, subscripts of the f's that precede the comma refer to the β's; those following the comma refer to the λ's.

The system of equations that results for the f-functions is

$$L_1 f_{1,0} = F_0'^2 - 1 \tag{37a}$$

$$L_1 f_{0,1} = F_0' - 1 \tag{37b}$$

$$\left.\begin{aligned}
L_2 f_{2,0} &= F_0'^2 - 1 \\
L_2 f_{11,0} &= 2F_0' f_{1,0}' + (\beta_0 + 2) f_{1,0}'^2 - 3f_{1,0} f_{1,0}'' \\
L_2 f_{1,1} &= 2F_0' f_{0,1}' + 2(\beta_0 + 2) f_{1,0}' f_{0,1}' - 3(f_{1,0} f_{0,1}'' + f_{0,1} f_{1,0}'') + f_{1,0}' \\
L_2 f_{0,11} &= f_{0,1}' + (\beta_0 + 2) f_{0,1}'^2 - 3f_{0,1} f_{0,1}'' \\
L_2 f_{0,2} &= F_0' - 1
\end{aligned}\right\} \tag{38}$$

$$\left.\begin{aligned}
L_3 f_{3,0} &= F_0'^2 - 1 \\
L_3 f_{111,0} &= f_{1,0}'^2 + 2F_0' f_{11,0}' + 2(\beta_0 + 3) f_{1,0}' f_{11,0}' \\
&\quad - 3f_{1,0} f_{11,0}'' - 5f_{1,0}'' f_{11,0} \\
L_3 f_{12,0} &= 2F_0' f_{2,0}' + 2F_0' f_{1,0}' + 2(\beta_0 + 3) f_{1,0}' f_{2,0}' \\
&\quad - 3f_{1,0} f_{2,0}'' - 5f_{1,0}'' f_{2,0} \\
L_3 f_{11,1} &= 2f_{1,0}' f_{0,1}' + 2F_0' f_{1,1}' + 2(\beta_0 + 3)(f_{1,0}' f_{1,1}' + f_{0,1}' f_{11,0}') \\
&\quad - 3f_{1,0} f_{1,1}'' - 3f_{0,1} f_{11,0}'' - 5(f_{1,0}'' f_{1,1} + f_{0,1}'' f_{11,0}) + f_{11,0}' \\
L_3 f_{1,11} &= f_{0,1}'^2 + 2F_0' f_{0,11}' + 2(\beta_0 + 3)(f_{1,0}' f_{0,11}' + f_{0,1}' f_{1,1}') \\
&\quad - 3(f_{1,0} f_{0,11}'' + f_{0,1} f_{1,1}'') - 5(f_{1,0}'' f_{0,11} + f_{1,1} f_{0,1}'') + f_{1,1}' \\
L_3 f_{1,2} &= 2F_0' f_{0,2}' + 2(\beta_0 + 3) f_{1,0}' f_{0,2}' - 3f_{1,0} f_{0,2}'' - 5f_{1,0}'' f_{0,2} + f_{1,0}' \\
L_3 f_{2,1} &= 2F_0' f_{0,1}' + 2(\beta_0 + 3) f_{0,1}' f_{2,0}' - 3f_{0,1} f_{2,0}'' - 5f_{0,1}'' f_{2,0} + f_{2,0}' \\
L_3 f_{0,111} &= 2(\beta_0 + 3) f_{0,1}' f_{0,11}' - 3f_{0,1} f_{0,11}'' - 5f_{0,1}'' f_{0,11} + f_{0,11}' \\
L_3 f_{0,12} &= 2(\beta_0 + 3) f_{0,1}' f_{0,2}' - 3f_{0,1} f_{0,2}'' - 5f_{0,1}'' f_{0,2} + f_{0,2}' + f_{0,1}' \\
L_3 f_{0,3} &= F_0' - 1
\end{aligned}\right\} \tag{39}$$

Each of the f's satisfies the boundary conditions (eq. (36)).

ENERGY EQUATION

If T is the temperature of the fluid and the fluid properties are assumed constant, the energy equation is

$$\rho c_p\left(U \frac{\partial T}{\partial X} + V \frac{\partial T}{\partial Y}\right) = U \frac{dP}{dX} + k \frac{\partial^2 T}{\partial Y^2} + \sigma U^2 B_Y^2 + \rho\nu\left(\frac{\partial U}{\partial Y}\right)^2 \tag{40}$$

It is apparent that the temperature at the outer edge of the boundary layer is not constant but is influenced by the pressure gradient and Joule heating. In this situation boundary conditions are simplified by introducing the total enthalpy

$$h = c_p T + \frac{1}{2} U^2 \tag{41}$$

In terms of a dimensionless enthalpy ratio

$$H \equiv \frac{h - h_w}{h_e - h_w} \tag{42}$$

in which h_w and h_e are constants denoting the values of the enthalpy at the wall and external to the boundary layer, respectively, the energy equation becomes

$$U \frac{\partial H}{\partial X} + V \frac{\partial H}{\partial Y} = \frac{\nu}{Pr} \frac{\partial^2 H}{\partial Y^2} - \frac{1 - Pr}{2Pr} \frac{\nu}{h_e - h_w} \frac{\partial^2 U^2}{\partial Y^2} \tag{43}$$

The right side of this equation vanishes in the free stream, and since the left side represents a derivative along a streamline, H (or h) is constant there. Physically, this means that any kinetic energy lost by the fluid in the free stream is returned to it as Joule heat.

In certain circumstances the nonhomogeneous term of equation (43) has little influence in the boundary layer. Obviously, if the Prandtl number is near 1, it is small.

For mercury ($Pr = 0.025$) the term is negligible for velocities up to 110 meters per second for the maximum temperature difference (between melting point and boiling point) of 400° C and is negligible for velocities up to 28 meters per second for a temperature difference of 25° C. If a hot gas is being considered, it is found that, when the fluid has progressed far enough from the stagnation point for the nonhomogeneous term to be important, the static temperature has decreased to a value at which the conductivity is, perhaps, an order of magnitude lower than its stagnation value. In such a case the assumption of constant conductivity made here is no longer valid, and, in fact, the magnetohydrodynamic body term may no longer be significant.

When the Prandtl number is set equal to unity in equation (43) and the boundary-layer transformations (22) and (25) are applied, the equation becomes

$$H_{\eta\eta} + FH_{\eta} = 2\xi(F_{\eta}H_{\xi} - F_{\xi}H_{\eta}) \tag{44}$$

This equation has been derived in reference 10 and, as shown (ref. 10), it can be resolved into a set of ordinary differential equations by expanding H in ξ:

$$H = H_0(\eta) + H_1(\eta)\xi + H_2(\eta)\xi^2 + \ldots \tag{45}$$

If an operator D_n is defined as

$$D_n \equiv \frac{d^2}{d\eta^2} + F_0 \frac{d}{d\eta} - 2nF_0' \tag{46}$$

then for each H_n there is an equation

$$D_n H_n = R_n \tag{47}$$

with the boundary conditions

$$\left.\begin{aligned} &H_0(0) = 0 && H_0(\infty) = 1 \\ &H_n(0) = H_n(\infty) = 0 && n \geq 1 \end{aligned}\right\} \tag{48}$$

The first two of these boundary conditions satisfy definition (42) at the surface and at the edge of the boundary layer. The conditions on the $H_n(n \geq 1)$ require them to vanish both at the surface and at the edge of the boundary layer, so that they are correction functions to H_0 that operate only with the boundary layer. The R_n are

$$\left.\begin{aligned} R_0 &= 0 \\ R_1 &= -3F_1H_0' \\ R_2 &= 2F_1'H_1 - 3F_1H_1' - 5F_2H_0' \\ R_3 &= 2F_2'H_1 - 3F_1H_2' + 4F_1'H_2 - 5F_2H_1' - 7F_3H_0' \end{aligned}\right\} \tag{49}$$

In order to derive equations defining universal functions, it is necessary to make the further expansions

$$\left.\begin{aligned}
H_1 &= \beta_1 g_{1,0} + \lambda_1 g_{0,1} \\
H_2 &= \beta_2 g_{2,0} + \beta_1^2 g_{11,0} + \beta_1\lambda_1 g_{1,1} + \lambda_1^2 g_{0,11} + \lambda_2 g_{0,2} \\
H_3 &= \beta_3 g_{3,0} + \beta_1\beta_2 g_{12,0} + \beta_1^3 g_{111,0} + \beta_2\lambda_1 g_{2,1} + \beta_1^2\lambda_1 g_{11,1} \\
&\quad + \beta_1\lambda_1^2 g_{1,11} + \beta_1\lambda_2 g_{1,2} + \lambda_1^3 g_{0,111} + \lambda_1\lambda_2 g_{0,12} + \lambda_3 g_{0,3}
\end{aligned}\right\} \tag{50}$$

The g's are determined by

$$\left.\begin{aligned}
D_0 H_0 &= 0 \\
D_1 g_{1,0} &= -3f_{1,0}H_0' \\
D_1 g_{0,1} &= -3f_{0,1}H_0'
\end{aligned}\right\} \tag{51}$$

$$\left.\begin{aligned}
D_2 g_{2,0} &= -5f_{2,0}H_0' \\
D_2 g_{11,0} &= 2f_{1,0}' g_{1,0} - 3f_{1,0} g_{1,0}' - 5f_{11,0}H_0' \\
D_2 g_{1,1} &= 2f_{0,1}' g_{1,0} + 2f_{1,0}' g_{0,1} - 3f_{0,1} g_{1,0}' - 3f_{1,0} g_{0,1}' - 5f_{1,1}H_0' \\
D_2 g_{0,11} &= 2f_{0,1}' g_{0,1} - 3f_{0,1} g_{0,1}' - 5f_{0,11}H_0' \\
D_2 g_{0,2} &= -5f_{0,2}H_0'
\end{aligned}\right\} \tag{52}$$

$$\left.\begin{aligned}
D_3 g_{3,0} &= -7f_{3,0}H_0' \\
D_3 g_{12,0} &= 2f_{2,0}' g_{1,0} - 3f_{1,0} g_{2,0}' + 4f_{1,0}' g_{2,0} - 5f_{2,0} g_{1,0}' - 7f_{12,0}H_0' \\
D_3 g_{111,0} &= 2f_{11,0}' g_{1,0} - 3f_{1,0} g_{11,0}' + 4f_{1,0}' g_{11,0} - 5f_{11,0} g_{1,0}' - 7f_{111,0}H_0' \\
D_3 g_{2,1} &= 2f_{2,0}' g_{0,1} - 3f_{0,1} g_{2,0}' + 4f_{0,1}' g_{2,0} - 5f_{2,0} g_{0,1}' - 7f_{2,1}H_0' \\
D_3 g_{11,1} &= 2f_{11,0}' g_{0,1} - 3f_{0,1} g_{11,0}' + 4f_{0,1}' g_{11,0} - 5f_{11,0} g_{0,1}' + 2f_{1,1}' g_{1,0} \\
&\quad - 3f_{1,0} g_{1,1}' + 4f_{1,0}' g_{1,1} - 5f_{1,1} g_{1,0}' - 7f_{11,1}H_0' \\
D_3 g_{1,11} &= 2f_{0,11}' g_{1,0} - 3f_{1,0} g_{0,11}' + 4f_{1,0}' g_{0,11} - 5f_{0,11} g_{1,0}' + 2f_{1,1}' g_{0,1} \\
&\quad - 3f_{0,1} g_{1,1}' + 4f_{0,1}' g_{1,1} - 5f_{1,1} g_{0,1}' - 7f_{1,11}H_0' \\
D_3 g_{1,2} &= 2f_{0,2}' g_{1,0} - 3f_{1,0} g_{0,2}' + 4f_{1,0} g_{0,2} - 5f_{0,2} g_{1,0}' - 7f_{1,2}H_0' \\
D_3 g_{0,111} &= 2f_{0,11}' g_{0,1} - 3f_{0,1} g_{0,11}' + 4f_{0,1}' g_{0,11} - 5f_{0,11} g_{0,1}' - 7f_{0,111}H_0' \\
D_3 g_{0,12} &= 2f_{0,2}' g_{0,1} - 3f_{0,1} g_{0,2}' + 4f_{0,1}' g_{0,2} - 5f_{0,2} g_{0,1}' - 7f_{0,12}H_0' \\
D_3 g_{0,3} &= -7f_{0,3}H_0'
\end{aligned}\right\} \tag{53}$$

For each of the g's in equations (51) to (53) the boundary conditions (48) transform to $g(0) = g(\infty) = 0$.

COEFFICIENTS OF SERIES EXPANSIONS

In order to apply the solutions to the boundary-layer equations that have been developed in the preceding section, it is necessary to convert the outer-velocity and magnetic-field distributions in terms of X into the functions $\beta(\xi)$ and $\lambda(\xi)$ developed as power series. The outer-velocity distribution around a two-dimensional symmetric blunt body is represented by

$$U_e = U_1X + U_3X^3 + U_5X^5 + \ldots \tag{54}$$

It is evident from equation (17) that U_1, U_3, U_5, . . . depend upon the magnetic field as well as on the pressure gradient. The variable ξ is the integral

$$\xi \equiv \frac{1}{\nu}\int_0^X U_e\,dX = Re_1\left(\frac{X}{L}\right)^2\left[1 + \frac{U_3L^2}{2U_1}\left(\frac{X}{L}\right)^2 + \frac{U_5L^4}{3U_1}\left(\frac{X}{L}\right)^4 + \ldots\right] \tag{55}$$

where

$$Re_1 \equiv \frac{U_1L^2}{2\nu} \tag{56}$$

Görtler shows that for the expansion

$$\beta(\xi) \equiv \frac{2\nu\xi U_e'}{U_e^2} = \beta_0 + \beta_1\xi + \beta_2\xi^2 + \ldots \tag{57}$$

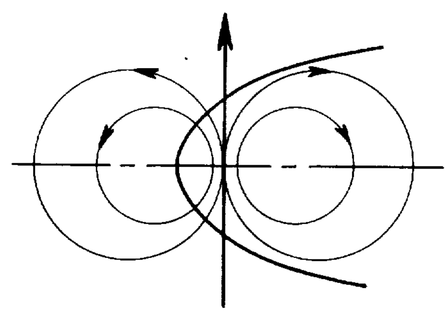
the coefficients β_i are

$$\left.\begin{aligned}
\beta_0 &= 1\\
\beta_1 &= Re_1^{-1}\,\frac{3}{2}\,\frac{U_3L^2}{U_1}\\
\beta_2 &= Re_1^{-2}\left(\frac{10}{3}\,\frac{U_5L^4}{U_1} - \frac{13}{4}\,\frac{U_3^2L^4}{U_1^2}\right)\\
\beta_3 &= Re_1^{-3}\left(\frac{21}{4}\,\frac{U_7L^6}{U_1} - 12\,\frac{U_3U_5L^6}{U_1^2} + \frac{27}{4}\,\frac{U_3^3L^6}{U_1^3}\right)\\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots
\end{aligned}\right\} \tag{58}$$

The coefficients in the expansion

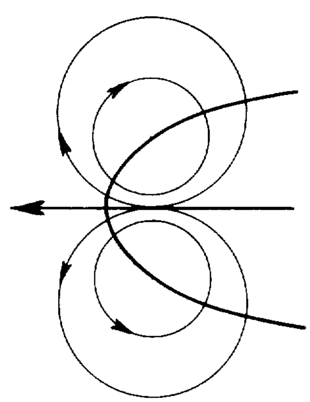
$$\lambda(\xi) \equiv 2\xi\sigma \frac{\nu B_Y^2}{\rho U_e^2} = \lambda_0 + \lambda_1\xi + \lambda_2\xi^2 + \ldots \tag{59}$$

depend explicitly upon the specification of the magnetic field as well as the velocity distribution. There are two symmetric types of magnetic field that may be associated with a blunt body. They correspond to antisymmetric and symmetric distributions of normal component on the body surface. An example of the first of these distributions is a two-dimensional dipole field with its axis perpendicular to the axis of the body (sketch (a)),



(a)

while an example of the second is a dipole field with its axis coincident with the body axis (sketch (b)).



(b)

In the first dipole orientation the normal component of the field B_Y is antisymmetric about the origin; therefore,

$$B_Y = B_1X + B_3X^3 + \ldots \tag{60}$$

Then

$$
\left.
\begin{aligned}
\lambda_0 &= 0 \\
\lambda_1 &= \frac{R_{m1}R_{h1}}{Re_1} \\
\lambda_2 &= \frac{R_{m1}R_{h1}}{Re_1^2}\, 2\left(\frac{B_3L^2}{B_1} - \frac{U_3L^2}{U_1}\right) \\
\lambda_3 &= \frac{R_{m1}R_{h1}}{Re_1^3}\left(2\,\frac{B_5L^4}{B_1} + \frac{B_3^2L^4}{B_1^2} - 2\,\frac{U_5L^4}{U_1} + 4\,\frac{U_3^2L^4}{U_1^2} - 5\,\frac{U_3L^2}{U_1}\,\frac{B_3L^2}{B_1}\right)
\end{aligned}
\right\}
\tag{61}
$$

where

$$R_{m1} = \sigma\mu U_1 L^2 \tag{62a}$$

$$R_{h1} = \frac{B_1^2}{\mu\rho U_1^2} \tag{62b}$$

The second sort of magnetic field is described by

$$B_Y = B_0 + B_2X^2 + B_4X^4 + \ldots \tag{63}$$

The expansion coefficients for $\lambda(\xi)$ are

$$
\left.
\begin{aligned}
\lambda_0 &= R_{m1}R_{h0} \\
\lambda_1 &= \frac{R_{m1}R_{h0}}{Re_1}\left(2\,\frac{B_2L^2}{B_0} - \frac{3}{2}\,\frac{U_3L^2}{U_1}\right) \\
\lambda_2 &= \frac{R_{m1}R_{h0}}{Re_1^2}\left(2\,\frac{B_4L^4}{B_0} + \frac{B_2^2L^4}{B_0^2} - 4\,\frac{U_3L^3}{U_1}\,\frac{B_2L^2}{B_0} - 5\,\frac{U_5L^4}{U_1} + \frac{11}{4}\,\frac{U_3^2L^4}{U_1^2}\right) \\
\lambda_3 &= \frac{R_{m1}R_{h0}}{Re_1^3}\left(-\frac{7}{4}\,\frac{U_7L^6}{U_1} + \frac{13}{2}\,\frac{U_3L^2}{U_1}\,\frac{U_5L^4}{U_1} - 4\,\frac{U_5L^2}{U_1}\,\frac{B_2L^2}{B_0} + 8\,\frac{U_3^2L^4}{U_1^2}\,\frac{B_2L^2}{B_0}\right. \\
&\quad \left. - \frac{21}{4}\,\frac{U_3^3L^6}{U_1^3} - 5\,\frac{U_3L^2}{U_1}\,\frac{B_4L^4}{B_0} - \frac{5}{2}\,\frac{U_3L^2}{U_1}\,\frac{B_2L^4}{B_0^2} + 2\,\frac{B_6L^6}{B_0} + 2\,\frac{B_2L^2}{B_0}\,\frac{B_4L^4}{B_0}\right)
\end{aligned}
\right\}
\tag{64}
$$

In these expressions the following definition has been used:

$$R_{h0} \equiv \frac{B_0^2}{\mu\rho U_1^2 L^2} \tag{65}$$

RESULTS OF COMPUTATIONS AND METHOD OF APPLICATION

The "universal" functions defined by equations (37) to (39) and (51) to (53) have been obtained by numerical integration for $Pr = 1$, $\beta_0 = 1$, and $\lambda_0 = 0$, 0.2, and 1. The computations were performed on an ERA 1103 digital computer. Double precision accuracy (20 significant figures) was used throughout the computations to ensure an adequate number of significant figures in the final results. Curves of the functions are presented in figures 1 to 3, and the values of the derivatives at the wall are listed in table I.

The functions $f_{i,j}$, $g_{i,j}$, and so forth are universal in the sense that they apply to any boundary layer started at the origin according to the values given to β_0 and λ_0. In the computations presented here β_0 is always 1, the value for a two-dimensional stagnation point; as a result, the functions computed apply to any symmetric, blunt body. As explained previously, the magnetic fields to which the functions apply are either symmetric or antisymmetric about the body axis and of such strength that the products of the dimensionless numbers are

$$R_{m1}R_{h1} = 0 \qquad \text{(antisymmetric field)}$$

$$R_{m1}R_{h0} = 0.2,\ 1 \qquad \text{(symmetric field)}$$

The practical results desired from an analysis of the boundary layer such as that presented here are the skin friction and the heat transfer along the surface of the body. The former, when expressed in terms of Görtler's variables, is

$$\left.\begin{aligned}
\tau &= \frac{\rho U_e^2 F_{\eta\eta}(\xi,0)}{\sqrt{2\xi}} \\
&= \frac{\rho U_e^2}{\sqrt{2\xi}}\Big\{F_0''(0) + \xi\left[\beta_1 f_{1,0}''(0) + \lambda_1 f_{0,1}''(0)\right] \\
&\quad + \xi^2\left[\beta_2 f_{2,0}''(0) + \beta_1^2 f_{11,0}''(0) + \beta_1\lambda_1 f_{1,1}''(0) + \lambda_1^2 f_{0,11}''(0) + \lambda_2 f_{0,2}''(0)\right] \\
&\quad + \xi^3\Big[\beta_3 f_{3,0}''(0) + \beta_1^3 f_{111,0}''(0) + \beta_1^2\lambda_1 f_{11,1}''(0) + \beta_1\lambda_1^2 f_{1,11}''(0) \\
&\quad + \beta_1\beta_2 f_{12,0}''(0) + \beta_1\lambda_2 f_{1,2}''(0) + \beta_2\lambda_1 f_{2,1}''(0) + \lambda_1^3 f_{0,111}''(0) \\
&\quad + \lambda_1\lambda_2 f_{0,12}''(0) + \lambda_3 f_{0,3}''(0)\Big] + \ldots\Big\}
\end{aligned}\right\} \tag{66}$$

The procedure for obtaining the skin friction in this form is to begin with the series expansions of the external velocity (eq. (54)) and the normal component of the magnetic field at the wall (eq. (60) or (63)). The series coefficients may be obtained either from a solution of the inviscid flow or from experimental data. From them one obtains $\xi(X)$ (eq. (55)), the β's (eq. (58)), and the λ's (eq. (61) or (64)). The second derivatives of F_0 and the f's, evaluated at the wall, are given in table I. These are all the numerical values required for an evaluation of skin friction according to equation (66).

Likewise the heat transfer is given by

$$q = -\frac{k(h_e - h_w)U_e}{c_p \sqrt{2\xi}\ \nu} H_\eta(0)$$

$$= -\frac{k(h_e - h_w)}{c_p \sqrt{2\xi}\ L} \frac{U_e L}{\nu} \left\{H_0'(0) + \xi\left[\beta_1 g_{1,0}'(0) + \lambda_1 g_{0,1}'(0)\right] + \ldots\right\}$$

The steps for computing this expression are the same as those for skin friction with the addition of the determination, either analytic or experimental, of the enthalpies h_e and h_w.

CONCLUDING REMARKS

A complete illustration of the use of the boundary-layer solutions generated in the foregoing discussion requires a knowledge of the external inviscid hydromagnetic flow about the blunt body. Such information has proved very difficult to obtain, both experimentally and analytically. Reference 13 discusses the flow about a circular cylinder from which a dipole magnetic field emanates. Even with the restrictions of very small field strength and low conductivity, this problem proves very difficult; therefore, while an overall drag coefficient can be extracted from the analysis, the flow is not described in detail.

Without investigating a particular configuration, it is possible to anticipate that the effect of the magnetic field upon the velocity profile, and thus upon separation, is likely to be much larger than its influence upon the temperature profile. This is shown, for example, by the magnitudes of the coefficients $f_{0,1}''(0)$, $f_{0,2}''(0)$, $f_{0,3}''(0)$, which are coefficients of purely magnetic terms; they are much larger than the corresponding g's that modify the heat transfer. Thus R. C. Meyer's conclusion, that the principal effect of the magnetic field on heat transfer occurs through the modification of the outer flow, is verified to terms of order three. However, because the effect of the magnetic field upon the separation point may be strong, the distance from the stagnation point over which this statement is true may, in general, be much less than the similarity solution studied by Meyer would indicate.

Lewis Research Center
National Aeronautics and Space Administration
Cleveland, Ohio, January 15, 1963

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TABLE I. - WALL VALUES OF DERIVATIVES OF UNIVERSAL

VELOCITY AND TEMPERATURE FUNCTIONS

[Blunt body (β_0 = 1); Prandtl number, 1.]

(a) Antisymmetric magnetic field (λ_0 = 0)

F_0''	1.23258 76568 12447	H_0'	0.57046 52525 020495
$f_{1,0}''$	.49384 05676 09244	$g_{1,0}'$	.06211 84021 43842
$f_{0,1}''$	.37484 85804 03703	$g_{0,1}'$	.04185 72212 39988
$f_{2,0}''$	.46454 08895 66132	$g_{2,0}'$	.06345 31227 78851
$f_{11,0}''$	-.07720 53879 57963	$g_{11,0}'$	-.02104 68203 50582
$f_{1,1}''$	-.12221 20640 84685	$g_{1,1}'$	-.03001 33056 75644
$f_{0,11}''$	-.04796 18827 56966	$g_{0,11}'$	-.01074 50574 18008
$f_{0,2}''$	.35754 69139 47016	$g_{0,2}'$	.04355 63514 48833
$f_{3,0}''$	.44238 30786 69987	$g_{3,0}'$	.06272 70799 22428
$f_{111,0}''$	.02241 54202 50688	$g_{111,0}'$	.00825 91634 91093
$f_{11,1}''$	.05514 64324 73607	$g_{11,1}''$	.01844 63956 69484
$f_{1,11}''$	.04461 07472 52109	$g_{1,11}'$	.01372 48880 50019
$f_{12,0}''$	-.13663 65852 25042	$g_{12,0}'$	-.03879 61227 09014
$f_{1,2}''$	-.11074 32208 63681	$g_{1,2}'$	-.02831 20924 24717
$f_{2,1}''$	-.10896 79251 52002	$g_{2,1}'$	-.02797 81250 04664
$f_{0,111}''$	.01190 31220 34659	$g_{0,111}'$	.00340 24247 40087
$f_{0,12}''$	-.08737 51499 90026	$g_{0,12}'$	-.02047 29739 25259
$f_{0,3}''$	.34348 98882 07614	$g_{0,3}'$	.04366 94245 21308

TABLE I. - Continued. WALL VALUES OF DERIVATIVES OF UNIVERSAL VELOCITY AND TEMPERATURE FUNCTIONS

[Blunt body ($\beta_0 = 1$); Prandtl number, 1.]

(b) Symmetric magnetic field ($\lambda_0 = 0.2$)

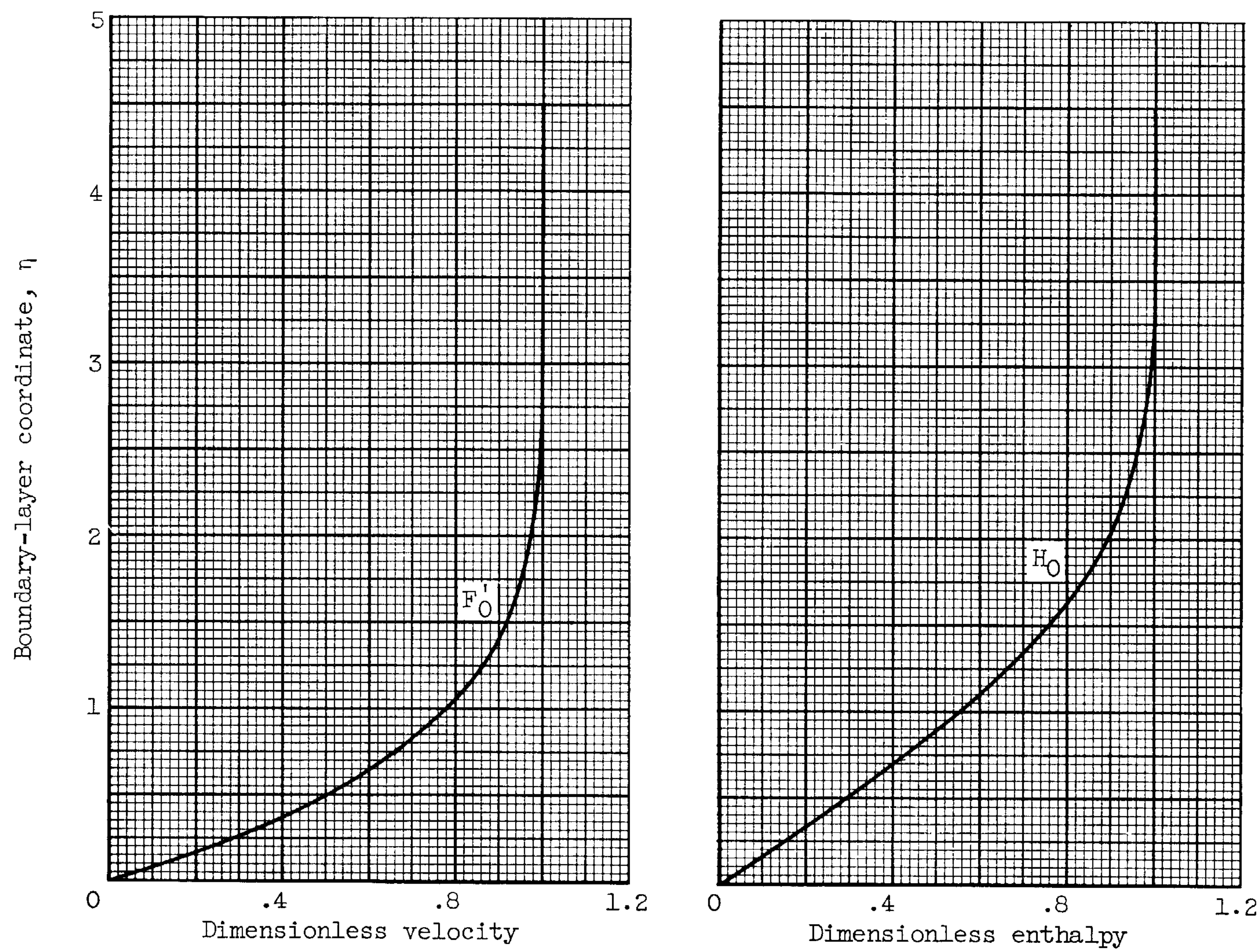
F''_0	1.31060 30130 24933	H'_0	0.57647 69026 36591
$f''_{1,0}$	.46802 25635 98567	$g'_{1,0}$	.05625 91084 43204
$f''_{0,1}$	.35579 85554 14712	$g'_{0,1}$	.03774 06599 44879
$f''_{2,0}$	.44189 18576 88439	$g'_{2,0}$	.05775 93910 52950
$f''_{11,0}$	-.06596 93769 68872	$g'_{11,0}$	-.01741 37963 78786
$f''_{1,1}$	-.10417 04610 74633	$g'_{1,1}$	-.02468 46998 47401
$f''_{0,11}$	-.04079 10050 43453	$g''_{0,11}$	-.00878 93370 36298
$f''_{0,2}$	.33951 48282 39504	$g''_{0,2}$	.03945 37867 57151
$f''_{3,0}$	.42189 40264 05088	$g''_{3,0}$	.05727 78011 53905
$f''_{111,0}$	.01711 37715 17790	$g'_{111,0}$	.00615 68555 14697
$f''_{11,1}$	.04206 27940 12478	$g'_{11,1}$	.01367 56044 66428
$f''_{1,11}$	.03398 29547 87311	$g'_{1,11}$	.01012 37664 34184
$f''_{12,0}$	-.11725 96879 10692	$g'_{12,0}$	-.03219 81602 91323
$f''_{1,2}$	-.09474 27295 01036	$g'_{1,2}$	-.02334 81249 98091
$f''_{2,1}$	-.09330 41605 65895	$g'_{2,1}$	-.02309 00178 47489
$f''_{0,111}$	.00905 40412 83318	$g'_{0,111}$	.00249 79370 09368
$f''_{0,12}$	-.07459 78709 01883	$g'_{0,12}$	-.01679 75760 76091
$f''_{0,3}$	.32687 29354 87525	$g'_{0,3}$	.03966 90973 26658

TABLE I. - Concluded. WALL VALUES OF DERIVATIVES OF UNIVERSAL VELOCITY AND TEMPERATURE FUNCTIONS

[Blunt body ($\beta_0 = 1$); Prandtl number, 1.]

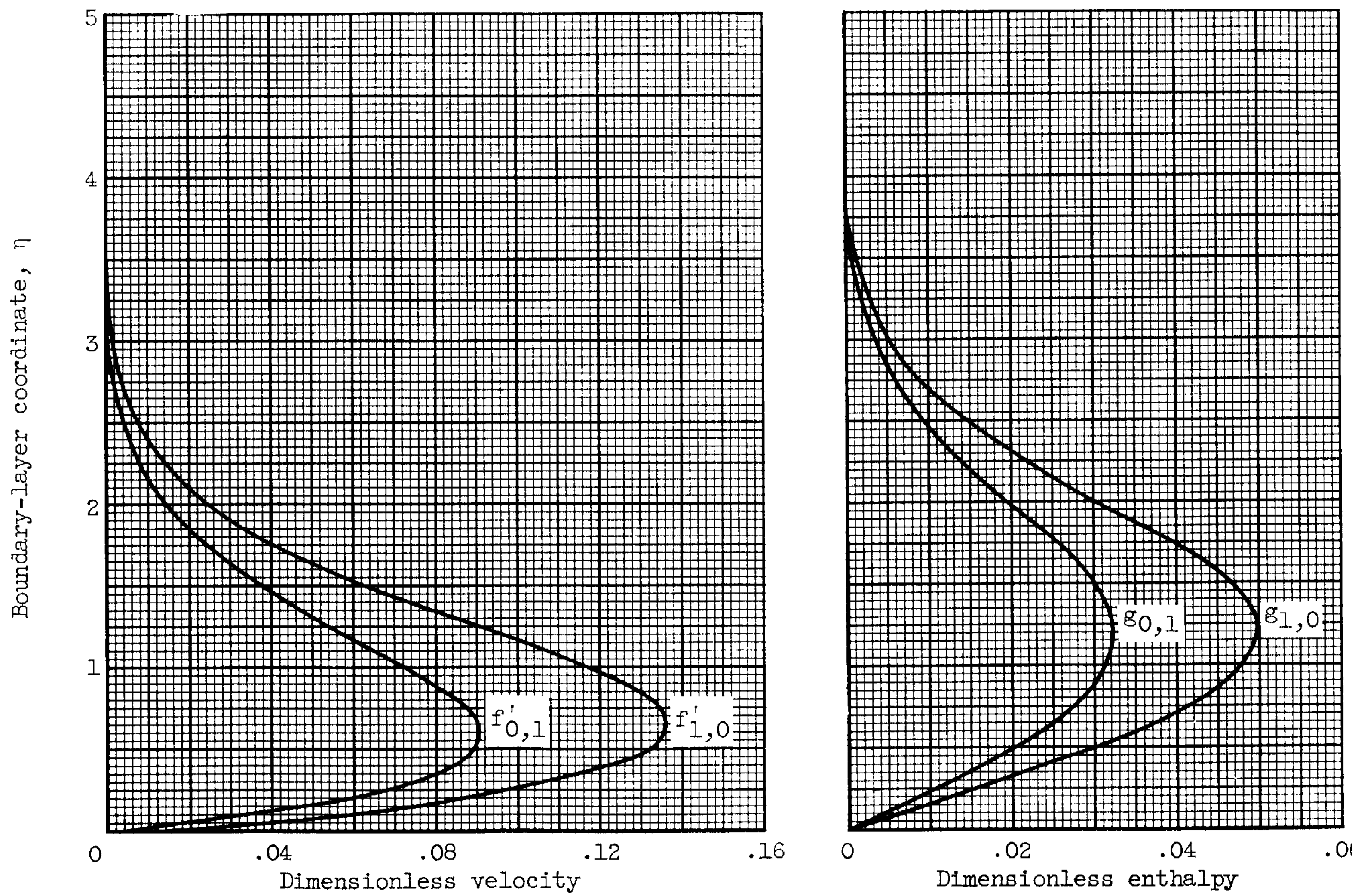
(c) Large symmetric magnetic field ($\lambda_0 = 1$)

F''_0	1.58533 06966 27504	H'_0	0.59534 63267 92991
$f''_{1,0}$	.39508 87794 71235	$g'_{1,0}$	.04087 91506 54384
$f''_{0,1}$	.29938 68944 84807	$g'_{0,1}$	.02705 75978 78049
$f''_{2,0}$	.37715 18405 62848	$g'_{2,0}$	.04267 69292 28947
$f''_{11,0}$	-.04035 05483 64961	$g'_{11,0}$	-.00957 43342 18736
$f''_{1,1}$	-.06320 33463 22469	$g'_{1,1}$	-.01331 71316 35810
$f''_{0,11}$	-.02457 34983 09369	$g''_{0,11}$	-.00466 04516 90449
$f''_{0,2}$	.28830 16653 76848	$g''_{0,2}$	.02871 96871 03897
$f''_{3,0}$	.36289 31275 81895	$g''_{3,0}$	.04277 06196 60540
$f''_{111,0}$	.00743 29922 53015	$g''_{111,0}$	.00246 38409 97704
$f''_{11,1}$	.01816 72938 22741	$g''_{11,1}$	.00536 96150 42514
$f''_{1,11}$	.01459 20154 82165	$g''_{1,11}$	.00390 67180 68167
$f''_{12,0}$	-.07276 07332 52263	$g''_{12,0}$	-.01790 15092 67214
$f''_{1,2}$	-.05817 50494 45701	$g''_{1,2}$	-.01271 87901 56523
$f''_{2,1}$	-.05744 65405 71772	$g''_{2,1}$	-.01260 68840 13622
$f''_{0,111}$	.00386 48724 45924	$g''_{0,111}$	.00094 87582 00696
$f''_{0,12}$	-.04549 40262 91466	$g''_{0,12}$	-.00900 12643 08112
$f''_{0,3}$	.27938 05871 65514	$g''_{0,3}$	.02915 50070 63649



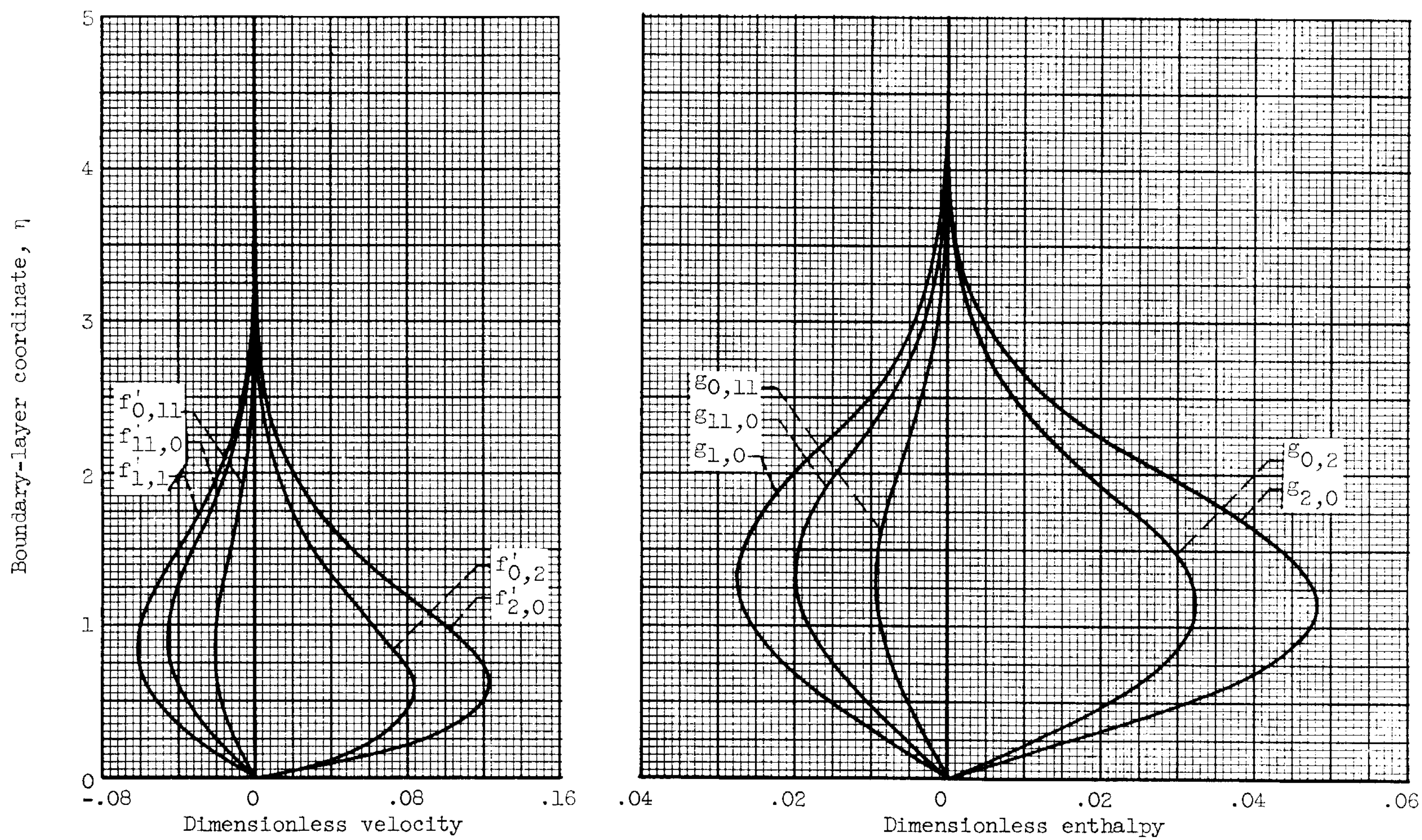
(a) Zero order.

Figure 1. - Universal velocity and enthalpy functions. Antisymmetric magnetic field ($\lambda_0 = 0$).



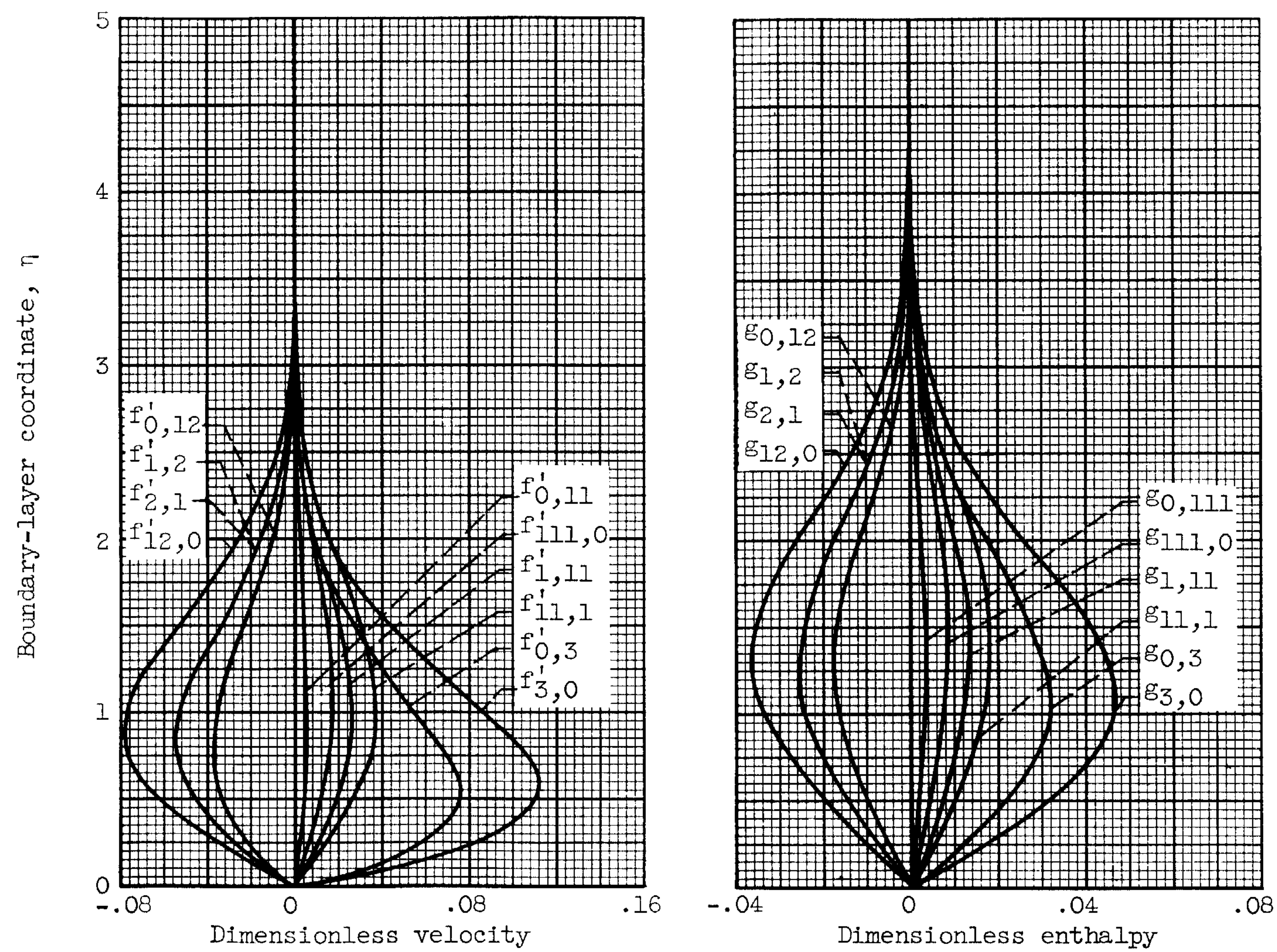
(b) First order.

Figure 1. - Continued. Universal velocity and enthalpy functions. Antisymmetric magnetic field ($\lambda_0 = 0$).



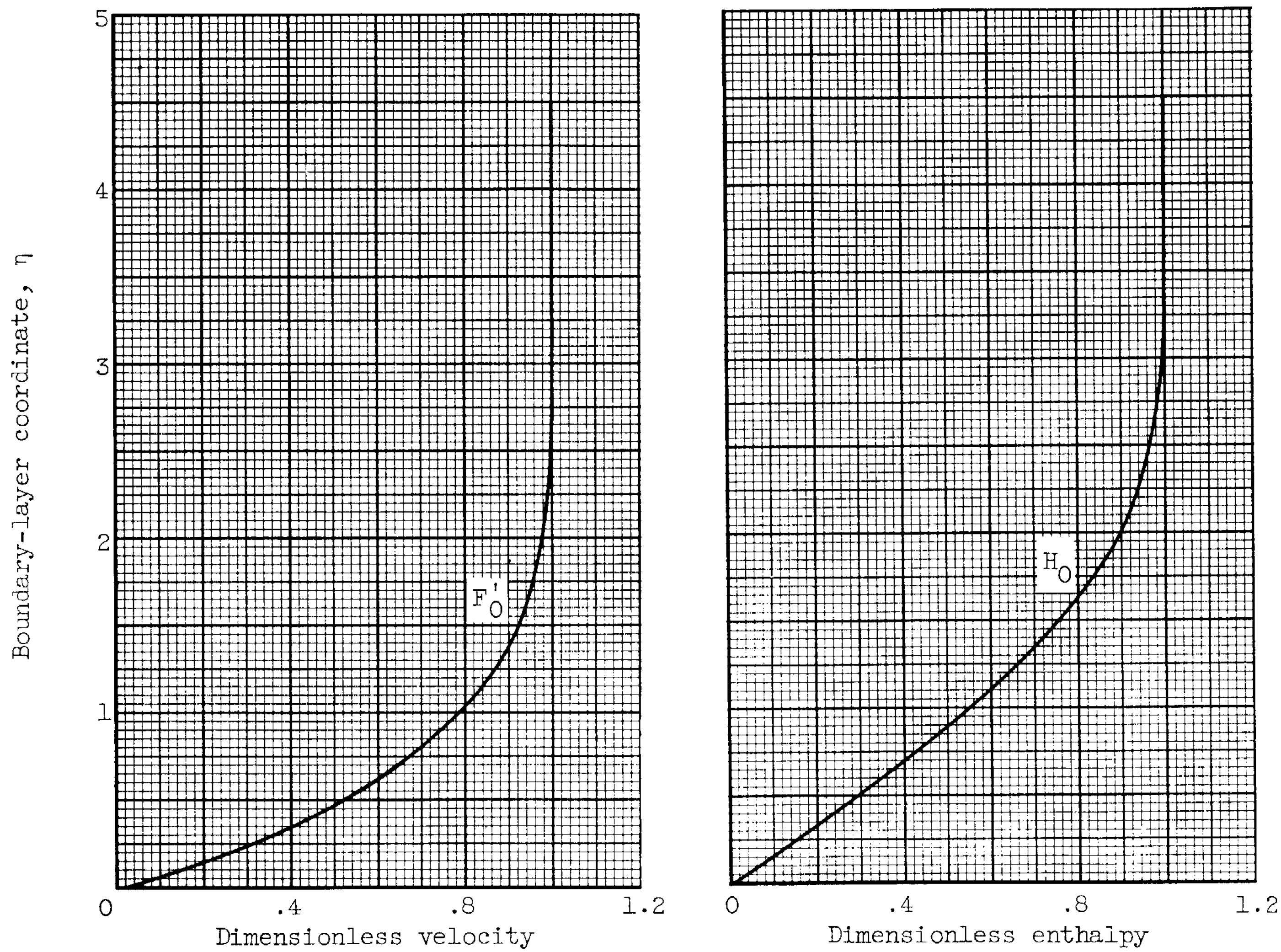
(c) Second order.

Figure 1. - Continued. Universal velocity and enthalpy functións. Antisymmetric magnetic field ($\lambda_0 = 0$).



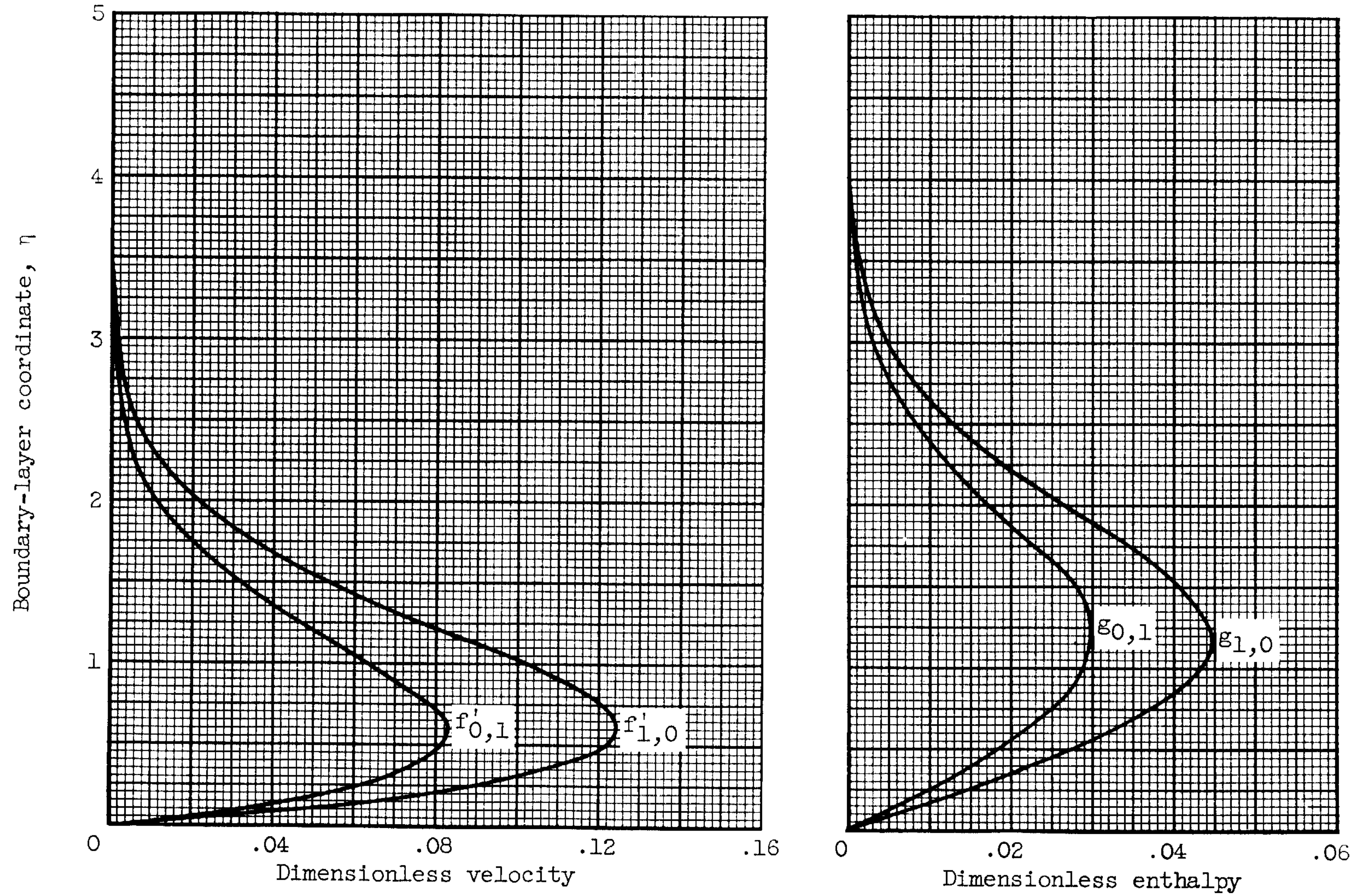
(d) Third order.

Figure 1. - Concluded. Universal velocity and enthalpy functions. Antisymmetric magnetic field ($\lambda_0 = 0$).



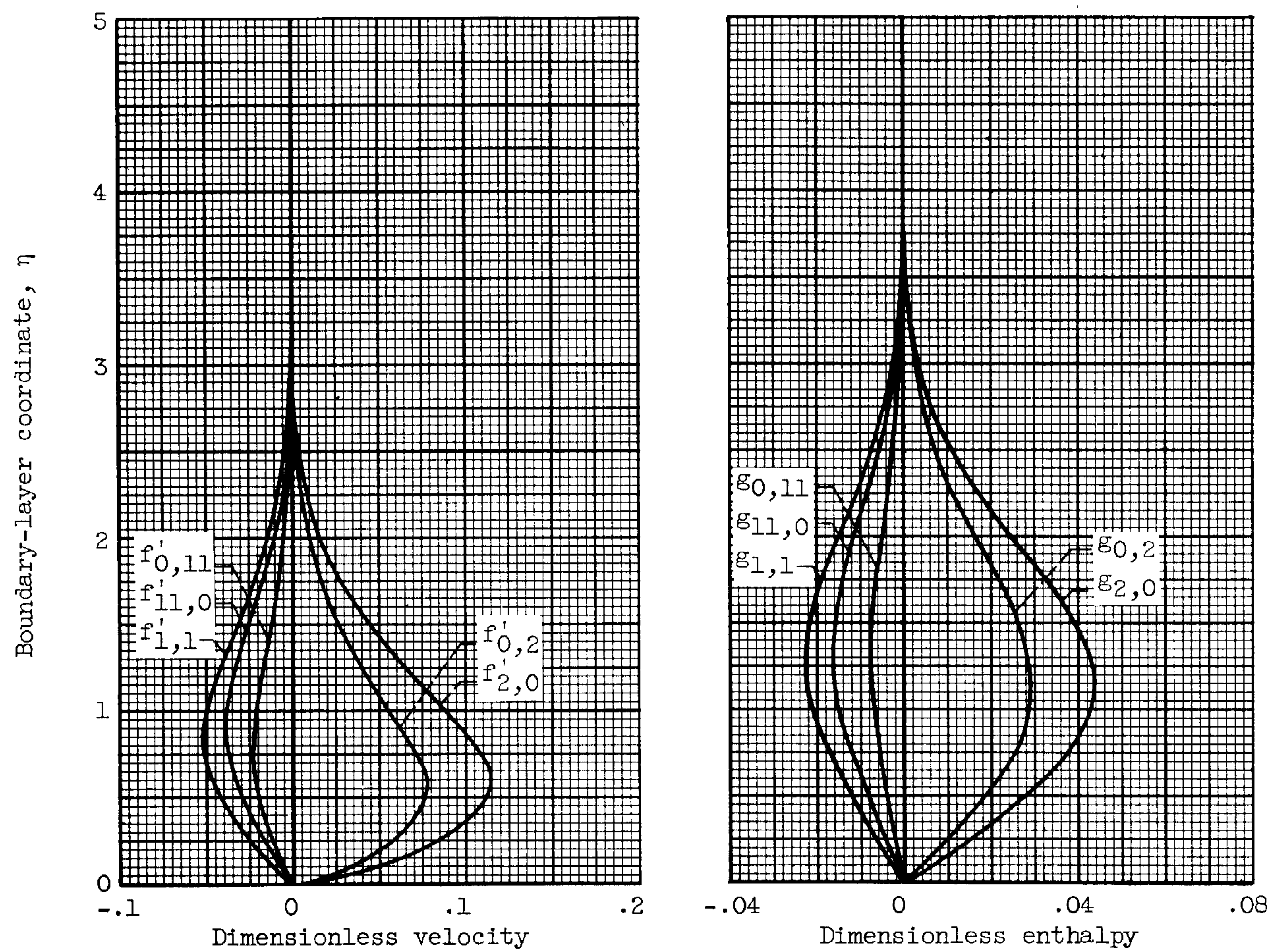
(a) Zero order.

Figure 2. - Universal velocity and enthalpy functions. Symmetric magnetic field ($\lambda_0 = 0.2$).



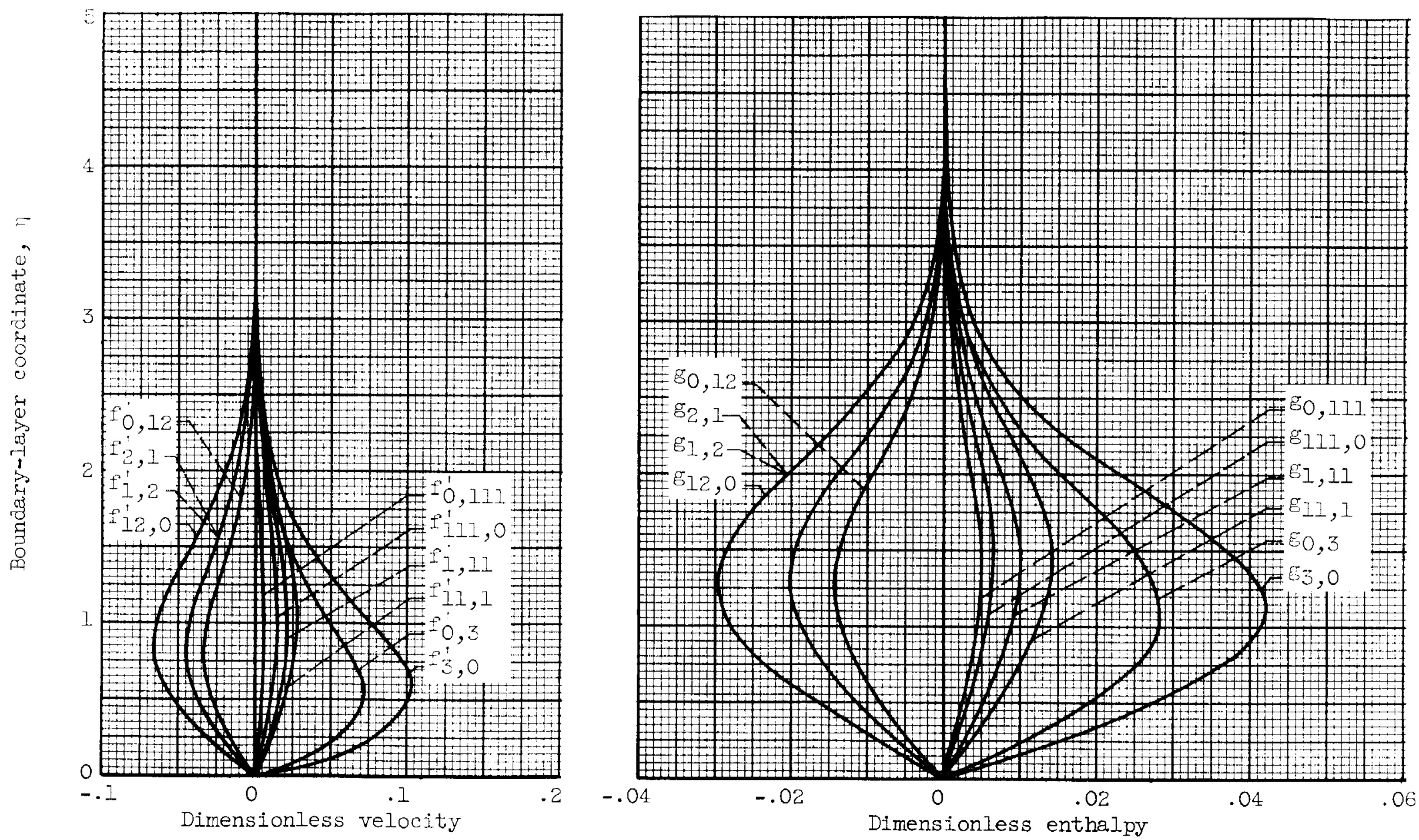
(b) First order.

Figure 2. - Continued. Universal velocity and enthalpy functions. Symmetric magnetic field ($\lambda_0 = 0.2$).



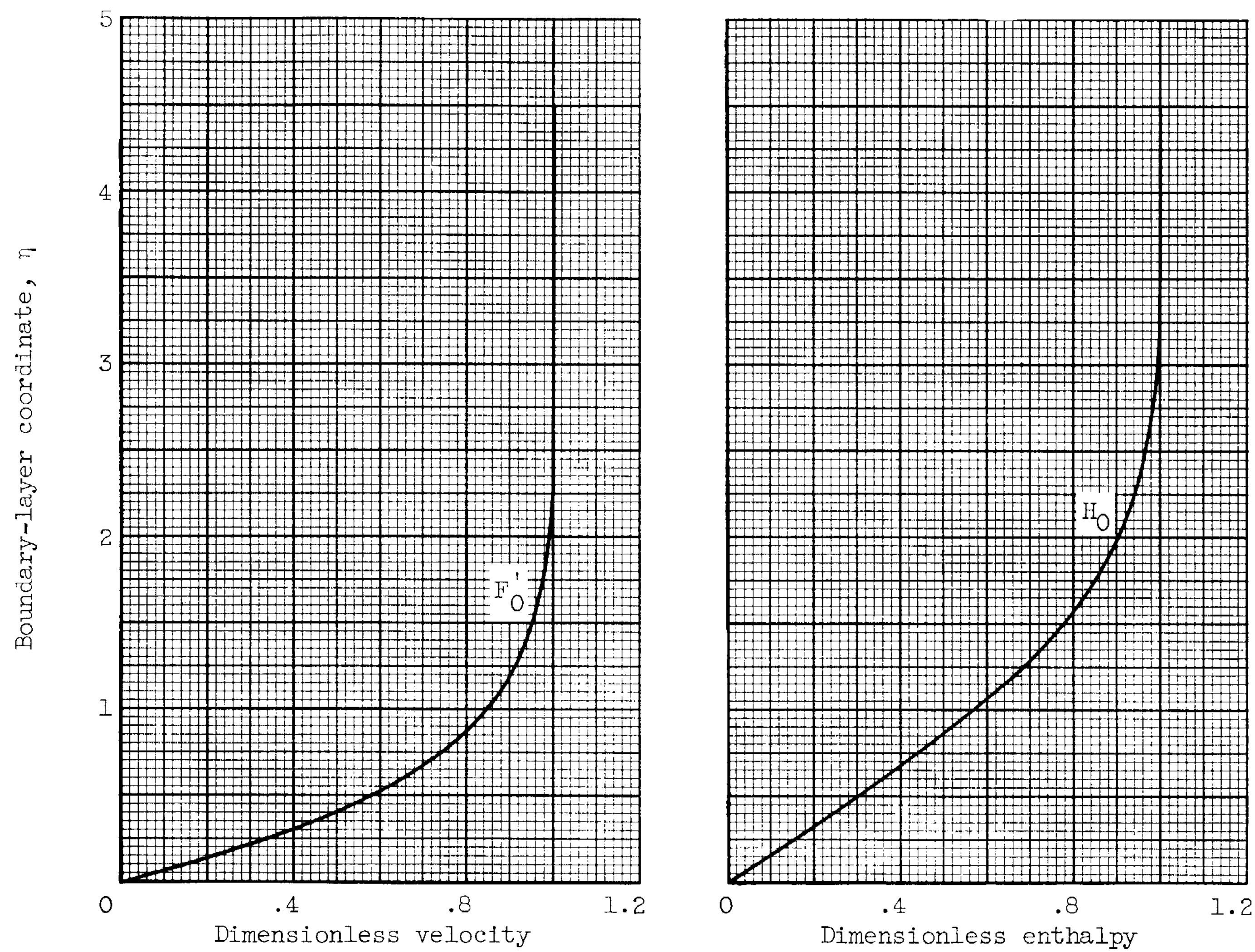
(c) Second order.

Figure 2. - Continued. Universal velocity and enthalpy functions. Symmetric magnetic field ($\lambda_0 = 0.2$).



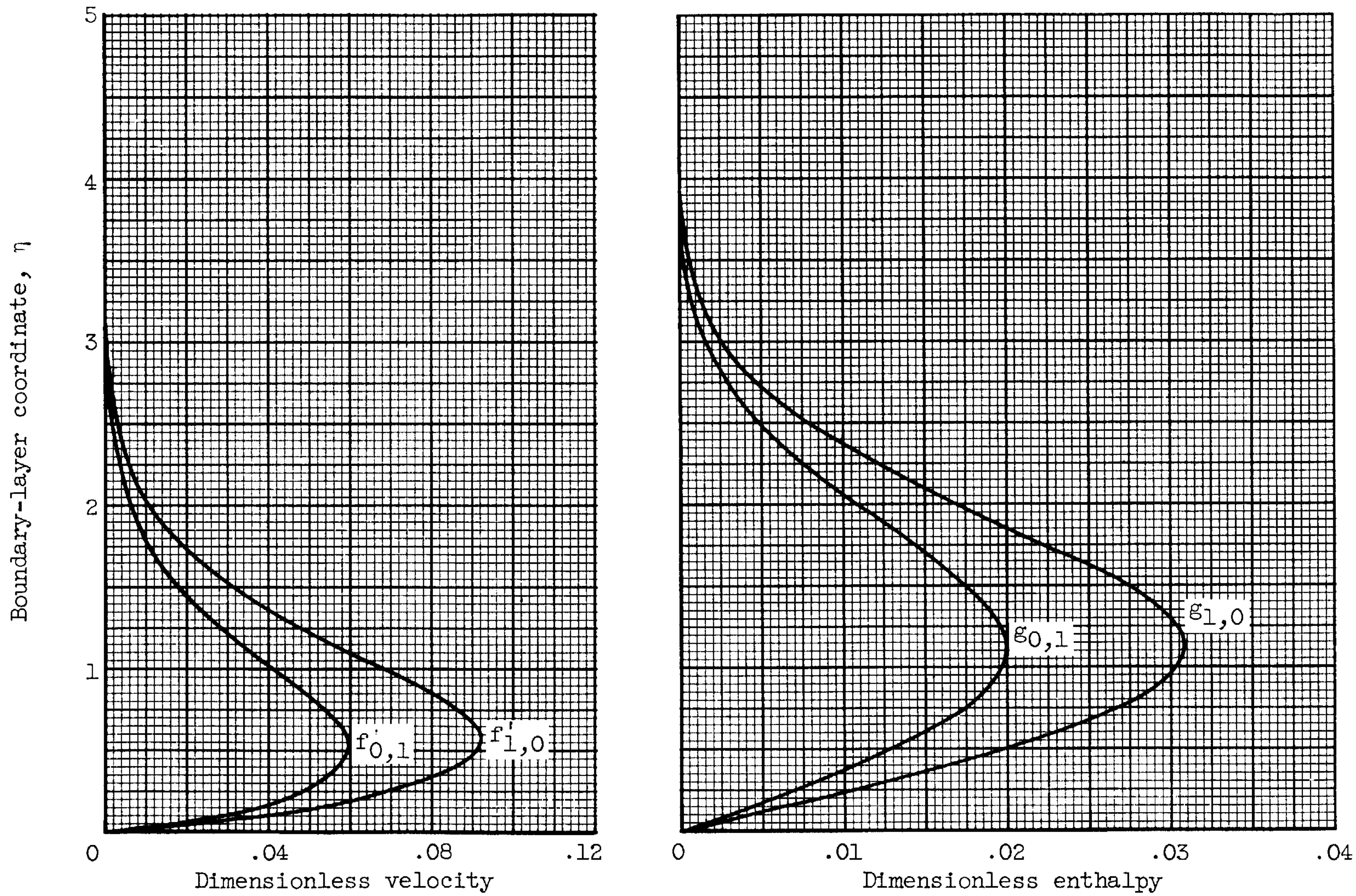
(d) Third order.

Figure 2. - Concluded. Universal velocity and enthalpy functions. Symmetric magnetic field ($\lambda_0 = 0.2$).



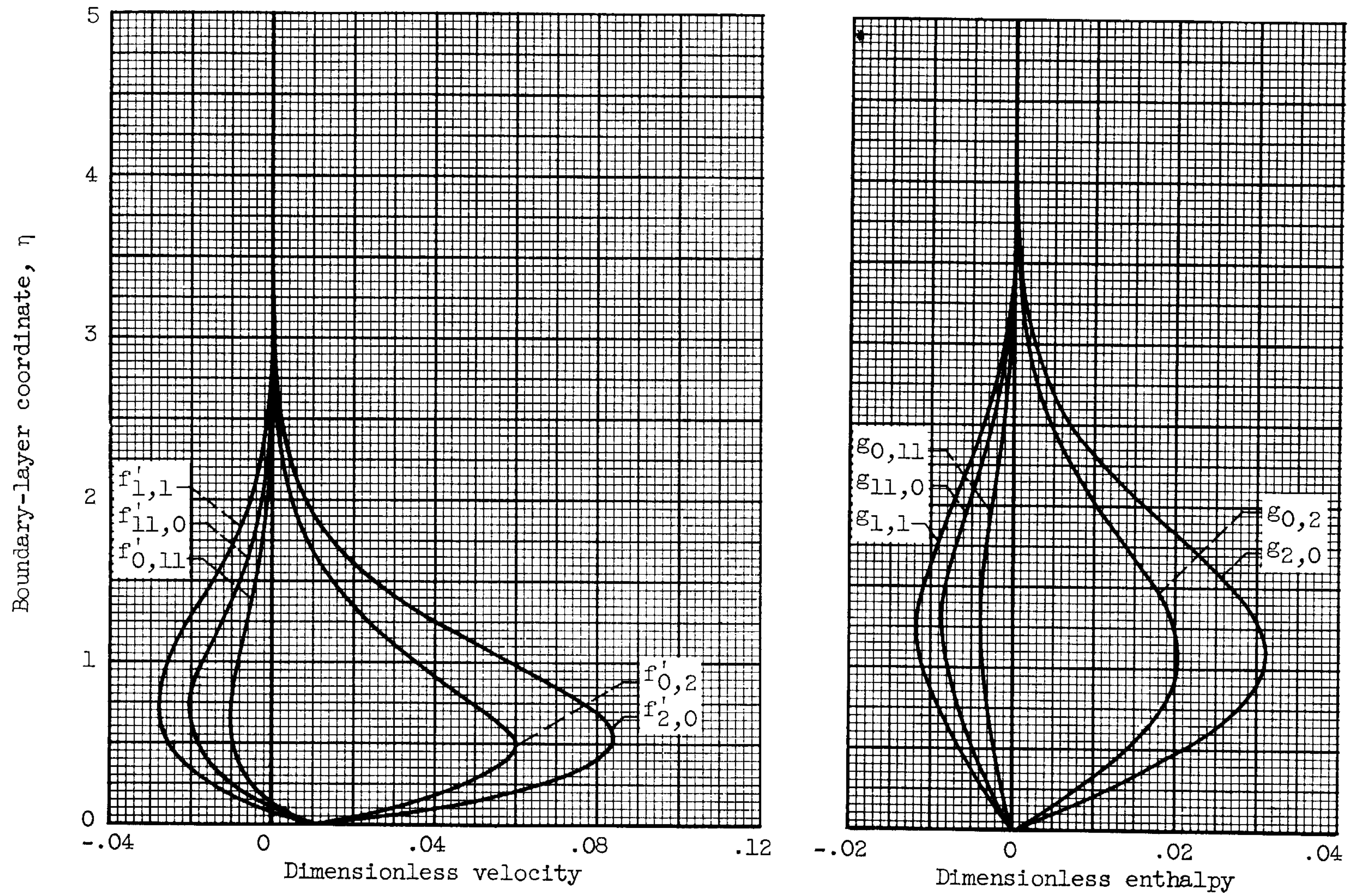
(a) Zero order.

Figure 3. - Universal velocity and enthalpy functions. Large symmetric magnetic field ($\lambda_0 = 1$).



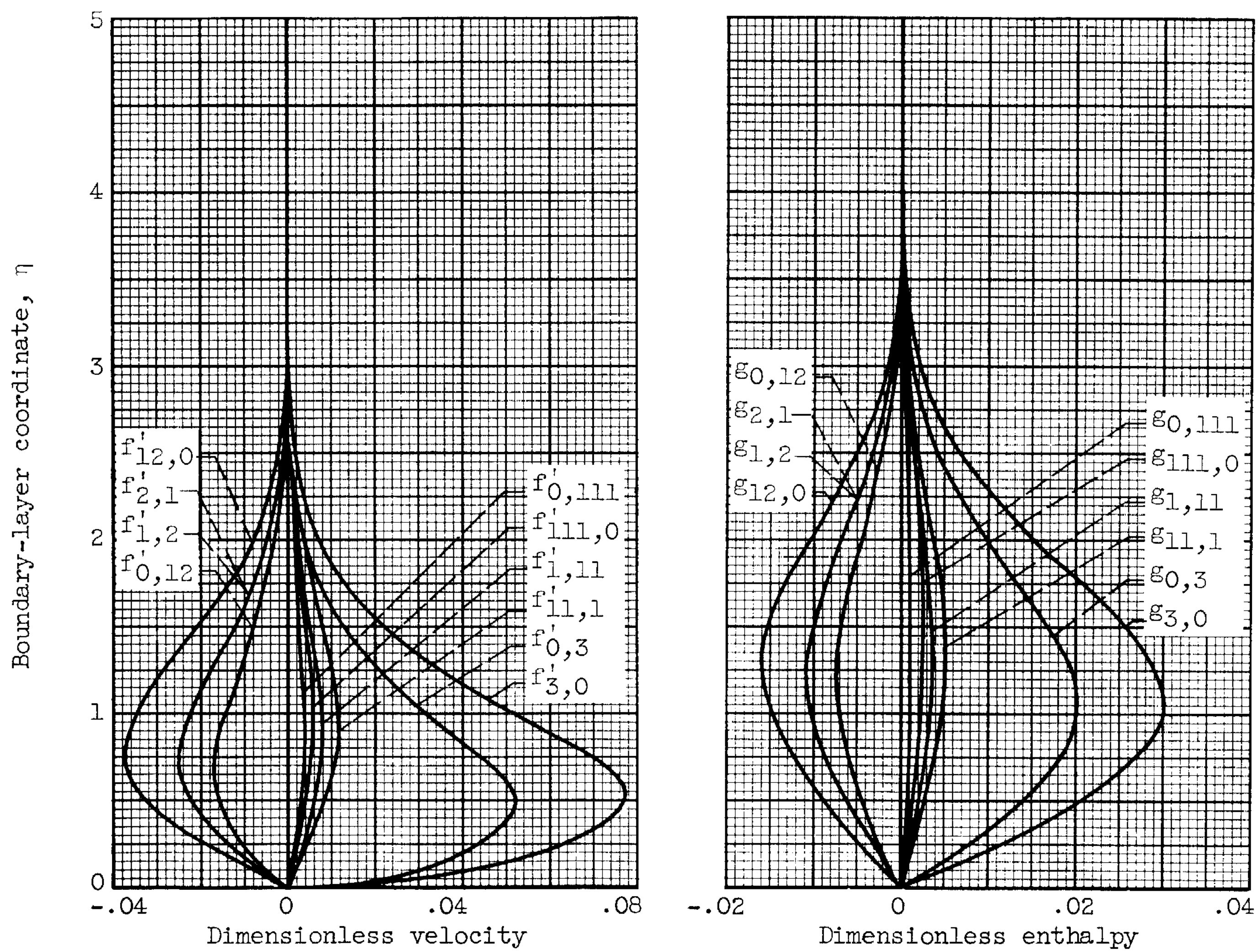
(b) First order.

Figure 3. - Continued. Universal velocity and enthalpy functions. Large symmetric magnetic field ($\lambda_0 = 1$).



(c) Second order.

Figure 3. - Continued. Universal velocity and enthalpy functions. Large symmetric magnetic field ($\lambda_0 = 1$).



(d) Third order.

Figure 3. - Concluded. Universal velocity and enthalpy functions. Large symmetric magnetic field ($\lambda_0 = 1$).

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